VIRTUAL MACHINING – THE MEAN OF MACHINE TOOLS VIRTUAL PROTOTYPING

prof. Ing. Peter Demeč, CSc. – doc. Ing. Kamil Madáč, CSc.
Technical University in Košice, Faculty of Mechanical Engineering, Slovakia

Abstract
Fabrication and testing of any technical prototype is realy expensive. That is why the intensive research works and verification of rapid prototyping are made for many approach. One of the method is virtual prototyping where mathematical method of prototyping is used. The paper deals with method of virtual machining on virtual machine tools, which make possible to find out the weak points during its design.

Keywords:
Virtual prototyping, machine tool, virtual machining

INTRODUCTION

Production and testing of any real technical product is time and money consuming process. That is why the research, development and verification of different approach of rapid prototyping are applied [1], [2], [3]). One of the possibilities is the virtual prototyping, the process of testing of designed product by means of its mathematical model. The paper will introduce the principle of virtual part machining on virtual machine tool which enable to find weak points of machine design during the design process.

THEORETICAL PRINCIPLES

There are basic mathematical relations developed in [4] for modeling of working accuracy of machine tools with serial kinematic structure in common form. Mathematical model of ideal machined surface is in matrix equation:

$$\mathbf{r}_0(t) = \left(\prod_{i=1}^{n-1} \mathbf{R}_{i,i-1}(t)\right).\,\mathbf{r}_n + \sum_{i=1}^{n-1}\left[\left(\prod_{j=1}^{i} \mathbf{R}_{j,j-1}(t)\right).\left(\mathbf{T}_{i+1,i}(t) + \mathbf{K}_{i+1,i}\right)\right] + \mathbf{T}_{10}(t) + \mathbf{K}_{10}\,, \tag{1}$$

where $\mathbf{r}_0(t)$ is position vector of active tool's point according to part's coordinate system

- $\mathbf{r}_n$ - position vector of active tool's point according to tool holder's coordinate,
- $\mathbf{R}_{i,i-1}(t)$ - Transformed matrix of rotation motion of modeled body T_i around of one coordinate axis of model body T_{i-1},
- $\mathbf{T}_{i+1,i}(t)$ - Transformed vector of straight motion of modeled body T_{i+1} in direction of one coordinate axis of model body T_i,
- $\mathbf{K}_{i+1,i}$ - vector of start position of coordinate system of model body T_{i+1} in coordinate system of model body T_i,
- t - time.

The final machining inaccuracy is determined by sum of part deformation due to production forces and position inaccuracy of all modeled bodies (machine nodes) from tool to machined part in coordinate system of machined part in time that in mathematical language could be write in a form

$$\Delta(t) = \Delta_0(t) + \sum_{i=1}^{n} \left\{\left[\prod_{j=1}^{i} \mathbf{R}_{j,j-1}(t)\right] . \left[\delta_i(t) + \varepsilon_i(t) . \mathbf{r}_i(t)\right]\right\}, \tag{2}$$

where deformation vector of machined part is

$$\Delta_0(t) = \delta_0(t) + \varepsilon_0(t) . \mathbf{r}_0(t) \tag{3}$$

And vector

$$\Delta_i(t) = \delta_i(t) + \varepsilon_i(t) . \mathbf{r}_i(t) \tag{4}$$

represents the final inaccuracy of active tool's point position caused by inaccuracy of model body T_i expressed in coordinate system model body itself.

The final inaccuracy of active tool's point position caused by inaccuracy of model body T_i, expressed in coordinate system of machined part model body could be write in a form

$$\Delta_{i,0}(t) = \left(\prod_{j=1}^{n} \mathbf{R}_{j,j-1}(t)\right) . \Delta_i(t), \tag{5}$$

The vector of linear inaccuracies of model body T_i is defined by relation

$$\delta_i(t) = [\,\delta_{xi}(t), \delta_{yi}(t), \delta_{zi}(t)\,]^T , \tag{6}$$

where $\delta_{xi}(t)$, $\delta_{yi}(t)$, $\delta_{zi}(t)$ are linear inaccuracies in directions of corresponding coordinate axis.

The matrix of angular inaccuracies of model body T_i is defined by relation

$$\varepsilon_i (t) = \begin{bmatrix} 0 & -\psi_i(t) & \upsilon_i(t) \\ \psi_i(t) & 0 & -\varphi_i(t) \\ -\upsilon_i(t) & \varphi_i(t) & 0 \end{bmatrix} \tag{7}$$

where $\varphi_i(t)$, $\upsilon_i(t)$, $\varphi_i(t)$ are angular inaccuracies (of rotation about axes X_i, Y_i, Z_i).

The position vector of active tool's point in coordinate system of model body T_i – vector $\mathbf{r}_i(t)$ in equations (2) and (4) is defined by relation

$$\mathbf{r}_i(t)=\left(\prod_{j=i}^{n-1} \mathbf{R}_{j+1,j}(t)\right).\mathbf{r}_n+\sum_{j=i}^{n-2}\left[\left(\prod_{k=i}^{j} \mathbf{R}_{k+1,k} (t)\right).(\mathbf{T}_{j+2,j+1}(t)+\mathbf{K}_{j+2,j+1})\right]+\mathbf{T}_{i+1,i}(t)+\mathbf{K}_{i+1,i} , \tag{8}$$

METHODOLOGY OF VIRTUAL MACHNINING

The described mathematical model could be used in virtual prototyping of designed machine tool for simulation of machined virtual part on virtual machine.

The whole process start with creating of computational model of designed machine. Then we determine:

- Start position of modeled bodies including their coordinate systems
- Motions of modeled bodies
- Transformed matrix and vectors of modeled bodies in common form

Next step is design of virtual model of machined part and its mathematical definition. This virtual model will be „machined" on computer model of researched machine tool with aim to find out the working inaccuracy with certain probability. Therefore we are proposing the model of machined part as the body of simple geometric form with dimensions close to maximum values suitable for the corresponding machine tool. The same is true for material of the machined part. We choose the material which needs the maximal cutting forces for corresponding machine tool.

Machining of modeled part on mathematical model of researched machine tools we are proposing on the same steps. It is suitable to model the machining of the surfaces that are typical for corresponding machine tool (for example rotational surfaces for the lathe or plane surfaces for the mill). The cut condition during the machining must correspond to " the hardest one" suitable for the machine tool. When conditions of the machining are done we must again redefine the transformed matrices and vectors in computational model of machine tool.

The process of virtual machining itself is suitable to realize in two stages. (In first stage we will not take into account the effect of cutting forces). By means of matrix equation (1), which in fact represents mathematical model of ideal machined surface in common form, we calculate for different values of time $\in \langle 0 ; T \rangle$ components pf position vectors $\mathbf{r}_i(t)$ according to (8) and $\mathbf{r}_0(t)$ according to (1). While the T is the time needed for complete surface machining with corresponded machining parameters (feed and spindle speed).

If the time t is changing continually then the graphs $\mathbf{r}_i(t)$ a $\mathbf{r}_0(t)$, are continuous too. Practically in computer modeling it is more suitable to change the time in some intervals Δt, that are finite and minimal towards to whole time T of surface machining of machined part.

In the second stage of virtual machining we include into calculations the effect of cutting forces, which creates some inaccuracies of positions of modeled bodies. We are able to describe their positions in any time $t \in \langle 0 ; T \rangle$ by equations (2) to (7). Step by step we can find the mathematical model of the real machined surface and get the following relations:

$f_{1i}(t) = \Delta_i(t)$ according to relation (4) – time relations of the final position inaccuracy of tool active point caused by position inaccuracy of modeled body T_i expressed in coordinate system of the same modeled body T_i . The number of that relations corresponds to the number of modeled bodies.;

$f_{2i}(t) = \Delta_{i,0}(t)$ according to relation (5) – time relations of the final position inaccuracy of tool active point caused by position inaccuracy of modeled body T_i expressed in coordinate system of the machined part. The number of those relations corresponds to the number of modeled bodies, too;

$f_3(t) = \Delta_0(t)$ according to relation (3) – time relations of the deformation of the machined part expressed in its coordinate system;

$f_4(t) = \Delta(t)$ according to relation (2) – time relations of the final inaccuracy of the machined part expressed in its coordinate system;.

Practical use of those relations is as follows:

- relations $f_{1i}(t) = \Delta_i(t)$ we use for calculation of relations $f_{2i}(t)$,
- relation $f_3(t) = \Delta_0(t)$ we use for calculation of relation $f_4(t)$,
- relations $f_{2i}(t) = \Delta_{i,0}(t)$ we use for calculation of relation $f_4(t)$, but we can use them to find out the ratio of modeled bodies to final inaccuracy of machined part.

We get the mathematical model of the real machined surface as the sun of relations values (1) a (2) in corresponding time t. Therefore it is important to quant the time in the same intervals Δt, for modeling of machining inaccuracy as well as in modeling of ideal tool trajectory. The real machined surface we can mathematically express by equation

$$f_5(t) = \mathbf{r}_0(t) + \Delta(t)\ . \tag{9}$$

PRACTICAL DEMONSTRATION

Now we can show the practical sample of virtual machining of virtual part on virtual machine tool – model of mill with vertical spindle axes. Virtual model of the machine tool is on the figure 1 and its calculation model is illustrated on figure 2. The mill is replaced by six modeled bodies: T_1 – table x, T_2 – table y, T_3 – bed, T_4 – column, T_5 – spindle box, T_6 – spindle. Machined part is replaced by model body T_0.

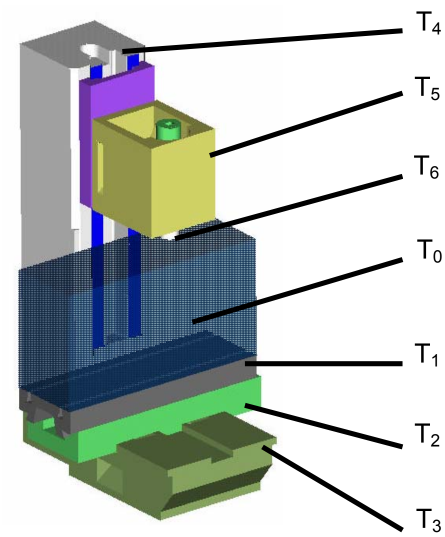

Figure 1
Virtual model of the machine

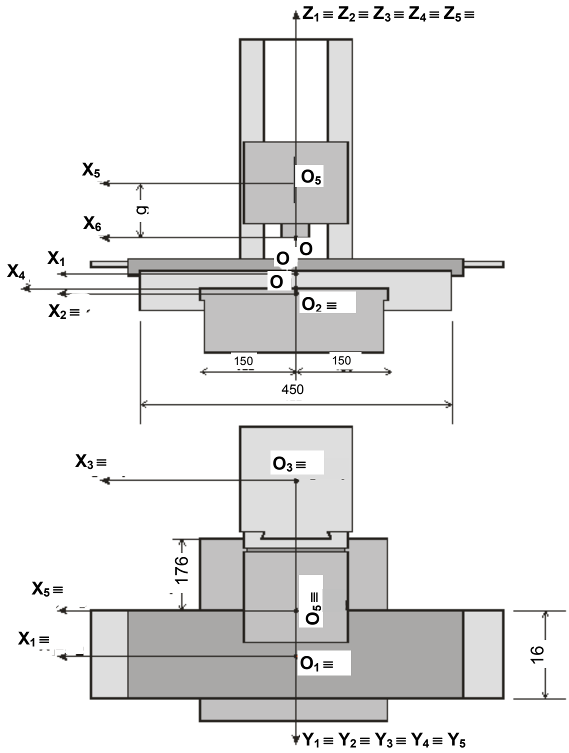

Figure 2
Calculation model of the machine

For virtual machining we use cylinder tool with diameter d_f = 10 mm (10x20 STN 22 2280 steel 19 856 teeth number z_f = 4), depth of cut h_f = 5 mm, wide of cut equal to tool diameter d_f. Spindle speed n_f = 500 min^{-1}, machining with motion of table X, the feed value f_{Xz} = 0,2 mm/tooth (400 $mm.min^{-1}$). For view in front of Mill the back part of upper surface is machined. We start to machine from right end of the machined part or left end position of table, see figure 3. The model of the machined part has dimensions L_o = 260 mm, B_o = 90 mm a H_o = 300 mm, material steel 11 500. Position of the part is symmetrical, while vertical planes of the part are identical with upper part of the table y.

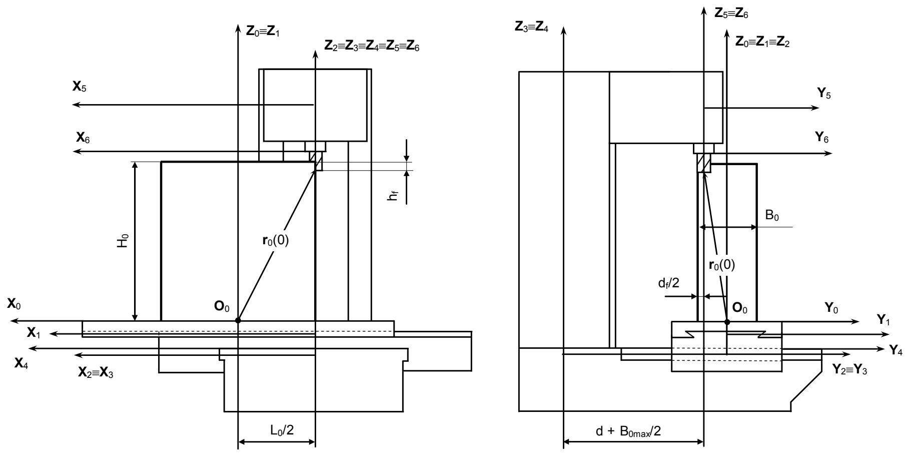

Figure 3
Dimensions and position of machined part at the beginning of virtual machining

With introduced conditions of the machining and after using of values for dimensions of machine tools and positions of machined part at machining start we can express the vectors $\mathbf{T}_{i,i-1}(t)$ a $\mathbf{K}_{i,I-1}$ in numerical forma s follows (values are in mm):

$$\left.\begin{aligned}
&\mathbf{T}_{65}(t) = \mathbf{T}_{54}(t) = \mathbf{T}_{43}(t) = \mathbf{T}_{32}(t) = \mathbf{T}_{10}(t) = \mathbf{0}\ ,\\
&\mathbf{T}_{21}(t) = [\ +\ s_2(t),\ 0,\ 0\ ]^T\ ,\\
&\mathbf{K}_{10} = [\ 0,\ 0,\ -\ 44\ ]^T\ ,\\
&\mathbf{K}_{21} = [\ -\ 130,\ 0,\ -\ 42\ ]^T\ ,\\
&\mathbf{K}_{32} = [\ 0,\ -\ 306,\ 0\ ]^T\ ,\\
&\mathbf{K}_{43} = [\ 0,\ 0,\ 6\ ]^T\ ,\\
&\mathbf{K}_{54} = [\ 0,\ 266,\ 515\ ]^T\ ,\\
&\mathbf{K}_{65} = [\ 0,\ 0,\ -\ 95\ ]^T\ .
\end{aligned}\right\} \qquad (10)$$

Position vector of tool's active point in coordinate system of the model body T_6 is

$$\mathbf{r}_n = \mathbf{r}_6 = [\ 0,\ 0,\ -45\ ]^T\ . \qquad (11)$$

Transformation vector $\mathbf{T}_{21}(t)$ we can refine by means of known feed value v_{fXmin} = 400 $mm.min^{-1}$. The trajectory of modeled body T_2, run in time t [s] is

$$s_2(t) = v_{fXmin}\ .\ t\ /\ 60 = 400\ t\ /\ 60\ , \qquad (12)$$

and the vector $\mathbf{T}_{21}(t)$ we can express as time dependence

$$\mathbf{T}_{21}(t) = [\ 400\ t\ /\ 60,\ 0,\ 0\ ]^T\ . \qquad (13)$$

Because the rotation motions of the modeled bodies was not presented, therefore transformation matrices of motions are unit, then

$$\mathbf{R}_{65}(t) = \mathbf{R}_{54}(t) = \mathbf{R}_{43}(t) = \mathbf{R}_{32}(t) = \mathbf{R}_{21}(t) = \mathbf{R}_{10}(t) = \mathbf{E}\ . \qquad (14)$$

For mathematical modeling of real machined surface we consider loadings as follows:
$F_X = F_Y$ = 4000 N a F_Z = 2000 N. We consider the weight of the machined part G_o = 540 N, spindle box with spindle G_v = 400 N, upper part of the table y G_1 = 150 N and bottom part of the table x G_2 = 220 N.

On the basis of input data the deformations of modeled bodies was calculated with exception of the bed that was considered as a rigid body. Some results of the numerical simulations are introduced on the figure 4

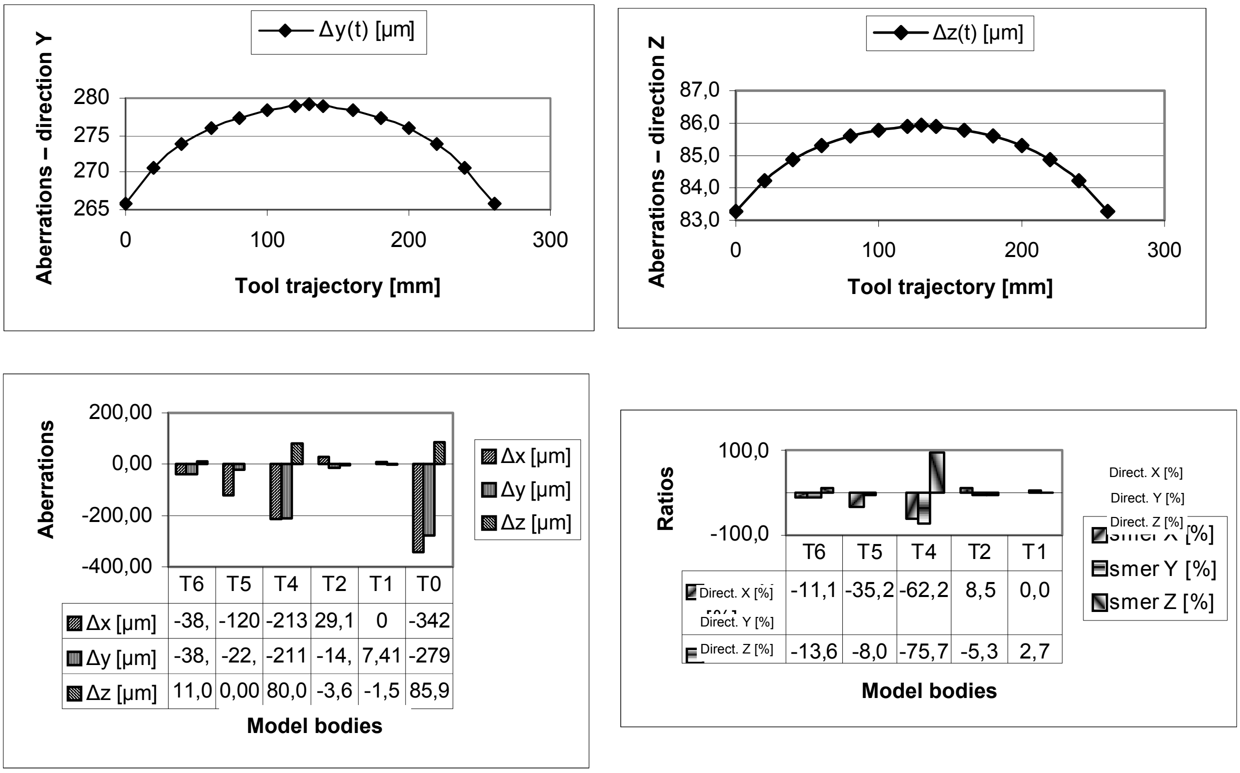

Figure 4
Sample of numerical simulation's results of virtual machining

CONCLUSION

The analyses of the results shows that the biggest inaccuracy of machining surface in Y and Z direction (the X direction is not interested because the tool does not create the final surface in this machining approach of machining) is generated in the middle position of part during the machining. It could be caused by the fact that in this position there is no compensation effect of upper table tilting due to compliance of contact surfaces of slides

By this approach of virtual machining of virtual part on virtual machine we can find the weak points of machine design during the design process and find the right changes in the design.

Note: The paper is the result of solution of grant project VEGA number 1/4162/07 Research of working accuracy of machine tool in the phase of its virtual prototyping.

REFERENCES

[1] Pilc, J., Stančeková, D., Mičietová, A., Salaj, J.: *Jednoúčelové stroje a výrobné linky*. Žilina, EDIS 2001.
[2] Pilc, J., Stančeková, D.: *Základy stavby obrábacích strojov*. Žilina, EDIS 2004.
[3] Poppeová, V., Čuboňová, N., Uríček, J., Kumičáková, D.: *Automatizácia strojárskej výroby*. Žilina, EDIS 2002.
[4] Demeč, P.: *Presnosť obrábacích strojov a jej matematické modelovanie*. Košice, VIENALA 2001.