Mathematical model for feed drive system in microscopic motion area

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ABSTRACT

Purpose: Our objective is to develop an entirely new the feed drive system with the accuracy of tens nanometers and the stroke of tens millimetres achieved by only one mechanism.

Design/methodology/approach: The dynamic driving torques can be measured when the sinusoidal waves of the microscopic displacement are applied to the AC servo motor. The mathematical model of the feed drive system that considers this dynamic behaviour is proposed.

Findings: We make hysteresis phenomenon clear in the range from 2 μm to 100 μm by the nonlinear spring characteristics. Taking the friction model into account, the driving torque can be formulated. The proposed friction model represents successfully the experimental results for the sinusoidal wave within the ranges of amplitude from 2 μm to 100 μm. In case that the amplitude is 1 μm, though the amplitudes of the driving torque are slightly different, the curve shows the similar shape.

Research limitations/implications: The results of this research covers the feed drive system with the AC servo motor and rolling guide. But, because the dynamic behaviour the rolling element was analyzed, it can be also applicable to the feed drive system with rolling element.

Practical implications: This paper presents more details of driving torque in microscopic motion area, so that the performance of the feed drive system with rolling elements can be improved.

Originality/value: The originality of this research project is to develop an entirely new feed drive system with the one mechanism and with the high accuracy.

Keywords: Machine tool; Feed drive system; Driving torque; Rolling element

1. Introduction

To attain the precision in order of nanometer level, the validity reason for the combination of the coarse motion and the fine motion is that the resolution of the coarse motion can only be obtained by about 1mm due to the effects of the nonlinear behaviors of the rolling elements with the coarse motion. But, the analysis on the static behaviors, that is the relation of the displacement and the force for the rolling elements has already been given. The resolutions of the rotary encoder necessary for controlling the AC servo motor and of the linear scale of measuring the position of the table have been improved significantly. Therefore, if the dynamic behaviors of rolling elements were modeled mathematically and the control system that considered the characteristics was designed, the feed drive system with the accuracy of tens nanometers and the stroke of tens millimeters would be achieved by only one mechanism without the fine table.

Most of the feed drive systems include only the coarse motion where the components consist of the AC servo motor, an amplifier and some rolling elements (ball-screw etc.). The effect of a nonlinearity of rolling elements is one of important factors...
for high motion accuracy. In the long history of the feed drive systems, though the analyses of static behavior have been done, the dynamic behavior has never been examined and the analyzed characteristics are the entire feed drive mechanism with the rolling elements. Our final objective is to develop an entirely new feed drive system with the accuracy of tens nanometers and the stroke of tens millimeters achieved by only one mechanism. In this study we will model the nonlinear behaviors of the feed drive system and investigate element which influences the motion accuracy of the feed drive system greatly. Concretely, analyses of the dynamic driving torque when the sinusoidal waves of microscopic displacement were input to the AC servo motor and the detail analyses have been done [1-12].

2. Experimental apparatus and modelling

2.1. Experimental apparatus

The overall view of the experimental apparatus is shown in Fig. 1. Our controlled object is an XY table system using many machine tools. This system consists of two feed drive systems (corresponding to each of the X and Y axes). Either of feed drive systems includes an AC servo motor and an amplifier, a ball screw and a table that supported by linear ball guides. The AC servo motor and the ball screw are connected by a coupling. Also, the feed drive system contains some rolling elements, the bearing for the AC servo motor, the ball screw, the bearing that supports the ball screw and the linear ball guide. The nonlinear behaviors of the rolling element cause the degeneracy of motion accuracy of the feed drive system in microscopic motion. Since there are some rolling elements in the feed drive system, we cannot specify which elements have strong influence on the motion accuracy. So, we focus on the dynamic behaviors of AC servo motor only.

Fig. 1. Experimental system

In this study, we examine only the characteristics of the AC servo motor controlled by a full closed feedback loop. In general the feedback signal is the rotational angle from a rotary encoder installed in the AC servo motor. But, it is convenient for us to define the linear displacement to express the dynamic behaviors of feed drive system in microscopic motion. We adopt the feedback signal converted from the rotational angle to the linear displacement. In our experiments apparatus the lead of ball screw is 5mm and the output of rotary encoder is 17 bits, so that 131.072 pulses are equivalent to one rotation. Thus, one pulse of the rotary encoder corresponds to the linear displacement of 38nm.

The electric current and the output velocity of the motor can be controlled successfully by a servo amplifier (controller). The rotary encoder acquires the displacement signal of the table and sends it to the control unit (the DSP board included in the PC). In this way, the feed drive system makes it possible to control the output displacement of the table as the controlled variable. The driving torque can be measured with an analog monitoring function installed in the servo amplifier. Through an analog monitoring terminal, the electric voltage signal can be measured as the driving torque, in which the rated torque of the motor corresponds to the electric voltage of one volt. Since the voltage signals are introduced into the DSP board through AD converters, all of data can be easily recorded simultaneously. In our experiments, it should be noted that the effects of the measurement noise on the driving torque can be reduced by a relaxation filter. The feed drive system can be operated at the control sampling period of 0.1ms, and all data can be recorded at the sampling period of 1ms after the system is triggered off the reference input.

### Table 1. Parameters of model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
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<td>$T_d$</td>
<td>s</td>
<td>$0.69 \times 10^3$</td>
</tr>
<tr>
<td>DA</td>
<td>V</td>
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<td>$K_v$</td>
<td>Nmm/A</td>
<td>637</td>
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<td>$J_m$</td>
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<td>$C_m$</td>
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<td>$T_f$</td>
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<tr>
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<td>$T_g$</td>
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</tr>
<tr>
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<td>0.001</td>
<td>$T_u$</td>
<td>Nmm/rad</td>
<td>Variable</td>
</tr>
</tbody>
</table>

2.2. Modelling

The control system of AC servo motor can be represented as shown in Fig. 2. Where, $r$ (mm) is the input displacement, $u$ (A) is the control input, $T$ (Nm) is the driving torque and finally the measured value of $T$ can be obtained through the low pass filter. $\theta$ (rad) is the angular displacement and $x$ (mm) is the output displacement. $K_p$ (1/m) is the displacement gain, DA (V) is the DA convertor ratio, $V_g$ (rad/s(V)) is the velocity reference gain, $K_{vp}$ (1/s) is the velocity loop gain, $K_v$ (Nms/rad) is the velocity gain, the relationship of $K_{vp}$ and $K_v$ is $K_v=J_m/K_{vp}$, $T_u$ (s) is the integral time of the velocity loop, $T_v$ (s) is the integral time, the relationship of $T_u$ and $T_v$ is $T_u=T_v$ $T_f$ (s) is the time constant of a torque filter. $T_f$ (s) is the time constant of an approximated current feedback loop. $J_m$ (kgm²) is the inertia of motor rotor. $C_m$ (Nms/rad) is the viscous damping coefficient of the motor bearing and $l$ (mm) is the lead of the ball screw.

Though $u$ is the input to the current loop in the amplifiers for AC servo motors, the detail information of the current loop is not clear to the public. In this study, the current loop is approximated to one order transfer function and these parameters were evaluated by the experiment. The unknown viscous damping coefficient $C_m$ in Fig. 2, can be determined by the torque to coincide with the experimental result in a circular motion of the motor.
we focus on the dynamic behaviors of AC servo motor only. which elements have strong influence on the motion accuracy. So, rolling elements in the feed drive system in microscopic motion. Since there are some of the rolling element cause the degeneracy of motion accuracy of the feed drive system contains some rolling elements, the bearing servo motor and the ball screw are.

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Fig. 2. Block diagram of feed drive system for microscopic motion

3. Friction model and dynamic behaviour of driving torque

3.1. Friction model

In our past studies, we clarified the following relationship of the output displacement and the driving torque. In case that the input displacement is less than 2 µm, the relationship becomes exactly linear. In the range from 2 µm to 100 µm, the relationship shows the nonlinear spring characteristics. In case that the input displacement is larger than 100 µm, for the simulation is affected by the Coulomb friction. In this study, we make hysteresis phenomenon clear in the range from 2 µm to 100 µm by the nonlinear spring characteristics. Taking the mathematical model into account, the friction torque can be expressed by

\[
dT_c = \frac{T_1}{\theta} \cdot \frac{1}{1 + \theta} \tag{1}
\]

T_c = 0

In this region, we considered that the Coulomb friction is not acted for the table system. The friction torque T_c is calculated by numerical integration of equation (1). In equation (1), T_1 has the gradient of friction torque for displacement area x_i, where \( \theta \) is the angular displacement for displacement area x_i, a is parameter that changes to represent the shape of friction torque.

3.2. Behaviour of driving torque

We applied the mathematical friction model to the block diagram in Fig. 2 and simulations were carried out for input frequency 0.1 Hz. The sampling period is 1 ms. Fig. 3 shows the relations between the output displacement and the driving torque for the sinusoidal input that the amplitude is fixed 10 µm. The parameters in this simulations are \( T_1 = 1.140 \ \text{N} \), \( x_i = 4.0 \ \text{µm} \) \( (\theta) = 5.03 \times 10^{-2} \ \text{rad} \) and \( a = 1.2 \). These parameters can be decided by the comparison of this simulation with the following experimental result.

In case that the nonlinear spring characteristics is not considered Fig. 3(a), the output displacement has inverse phase for the driving torque. On the other hand, in case that the nonlinear spring characteristics is considered Fig. 3(b), the output displacement has same phase and the driving torque is distorted curve. The amplitude of torque in Fig. 3(b) is 10 times larger for Fig. 3(a), The effects of the nonlinear spring characteristics depict that the driving torque is significantly large.

Fig. 3. Simulation results of driving torque and displacement measured a) without nonlinear spring, b) with nonlinear spring

Fig. 4 shows the simulation and experiment results for the driving torque. The parameters of mathematical friction model is decided by the comparison of the experimental result with the initial driving torque tuned by each experiment. In the Fig. 4, the proposed friction model represents successfully the experimental results for the sinusoidal wave where the amplitude are 10 µm and 20 µm. In case that the amplitude is 1 µm, though the amplitudes of the driving torque are slightly different, the curve shows the similar shape.
4. Conclusions

In this study, to develop the feed drive system with the accuracy of tens nanometers and the stroke of tens millimeters achieved by only one mechanism, we proposed the mathematical model represented the dynamic behaviour of feed drive system in microscopic motion area. By comparing the experimental result and simulation result, we summarize conclusions as follows.

- In case that the nonlinear spring characteristics is not considered, the amplitude of the driving torque is 100 times larger than the case that it is considered.
- The proposed friction model represents successfully the dynamic behaviour of the area that depends on the nonlinear spring characteristics.

References